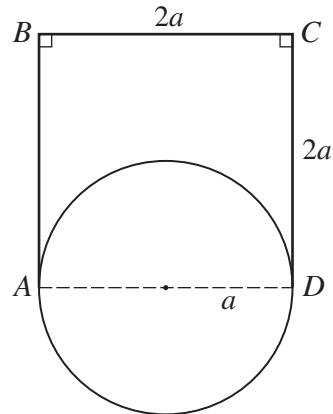




1

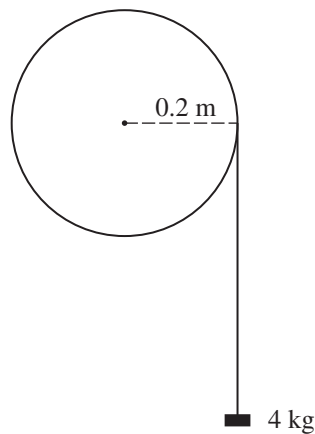


Three identical uniform rods,  $AB$ ,  $BC$  and  $CD$ , each of mass  $M$  and length  $2a$ , are rigidly joined to form three sides of a square. A uniform circular disc, of mass  $\frac{2}{3}M$  and radius  $a$ , has the opposite ends of one of its diameters attached to  $A$  and  $D$  respectively. The disc and the rods all lie in the same plane (see diagram). Find the moment of inertia of the system about the axis  $AD$ . [6]

- 2 The point  $O$  is on the fixed line  $l$ . The point  $A$  on  $l$  is such that  $OA = 3$  m. A particle  $P$  oscillates on  $l$  in simple harmonic motion with centre  $O$  and period  $\pi$  seconds. When  $P$  is at  $A$  its speed is  $12 \text{ m s}^{-1}$ . Find the speed of  $P$  when it is at the point  $B$  on  $l$ , where  $OB = 6$  m and  $B$  is on the same side of  $O$  as  $A$ . [4]

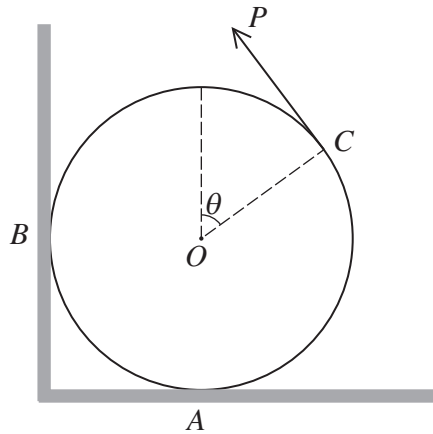
Find, correct to 2 decimal places, the time, in seconds, taken for  $P$  to travel directly from  $A$  to  $B$ . [4]

3



A uniform disc, of mass  $2$  kg and radius  $0.2$  m, is free to rotate in a vertical plane about a smooth horizontal axis through its centre. One end of a light inextensible string is attached to a point on the rim of the disc and the string is wound round the rim. The other end of the string is attached to a small block of mass  $4$  kg, which hangs freely (see diagram). The system is released from rest. During the subsequent motion, the block experiences a constant resistance to its motion, of magnitude  $R$  N. Given that the angular speed of the disc after it has turned through  $2$  radians is  $5 \text{ rad s}^{-1}$ , find  $R$  and the tension in the string. [9]

4



A uniform circular disc, with centre  $O$  and weight  $W$ , rests in equilibrium on a horizontal floor and against a vertical wall. The plane of the disc is vertical and perpendicular to the wall. The disc is in contact with the floor at  $A$  and with the wall at  $B$ . A force of magnitude  $P$  acts tangentially on the disc at the point  $C$  on the edge of the disc, where the radius  $OC$  makes an angle  $\theta$  with the upward vertical, and  $\tan \theta = \frac{4}{3}$  (see diagram). The coefficient of friction between the disc and the floor and between the disc and the wall is  $\frac{1}{2}$ . Show that the sum of the magnitudes of the frictional forces at  $A$  and  $B$  is equal to  $P$ . [2]

Given that the equilibrium is limiting at both  $A$  and  $B$ ,

(i) show that  $P = \frac{15}{34}W$ , [6]

(ii) find the ratio of the magnitude of the normal reaction at  $A$  to the magnitude of the normal reaction at  $B$ . [2]

5 Two uniform small smooth spheres  $A$  and  $B$ , of equal radii, have masses  $2m$  and  $m$  respectively. They lie at rest on a smooth horizontal plane. Sphere  $A$  is projected directly towards  $B$  with speed  $u$ . After the collision  $B$  goes on to collide directly with a fixed smooth vertical barrier, before colliding with  $A$  again. The coefficient of restitution between  $A$  and  $B$  is  $\frac{2}{3}$  and the coefficient of restitution between  $B$  and the barrier is  $e$ . After the second collision between  $A$  and  $B$ , the speed of  $B$  is five times the speed of  $A$ . Find the two possible values of  $e$ . [11]

6 A fair die is thrown until a 5 or a 6 is obtained. The number of throws taken is denoted by the random variable  $X$ . State the mean value of  $X$ . [1]

Find the probability that obtaining a 5 or a 6 takes more than 8 throws. [2]

Find the least integer  $n$  such that the probability of obtaining a 5 or a 6 in fewer than  $n$  throws is more than 0.99. [3]

7 A random sample of 10 observations of a normally distributed random variable  $X$  gave the following summarised data, where  $\bar{x}$  denotes the sample mean.

$$\Sigma x = 70.4 \quad \Sigma (x - \bar{x})^2 = 8.48$$

Test, at the 10% significance level, whether the population mean of  $X$  is less than 7.5. [7]

- 8 The lifetime, in years, of an electrical component is the random variable  $T$ , with probability density function  $f$  given by

$$f(t) = \begin{cases} Ae^{-\lambda t} & t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where  $A$  and  $\lambda$  are positive constants.

- (i) Show that  $A = \lambda$ . [2]

It is known that out of 100 randomly chosen components, 16 failed within the first year.

- (ii) Find an estimate for the value of  $\lambda$ , and hence find an estimate for the median value of  $T$ . [6]

- 9 For a random sample of 10 observations of pairs of values  $(x, y)$ , the equations of the regression lines of  $y$  on  $x$  and of  $x$  on  $y$  are

$$y = 4.21x - 0.862 \quad \text{and} \quad x = 0.043y + 6.36,$$

respectively.

- (i) Find the value of the product moment correlation coefficient for the sample. [3]

- (ii) Test, at the 10% significance level, whether there is evidence of non-zero correlation between the variables. [4]

- (iii) Find the mean values of  $x$  and  $y$  for this sample. [2]

- (iv) Estimate the value of  $x$  when  $y = 2.3$  and comment on the reliability of your answer. [2]

- 10 Customers were asked which of three brands of coffee,  $A$ ,  $B$  and  $C$ , they prefer. For a random sample of 80 male customers and 60 female customers, the numbers preferring each brand are shown in the following table.

	$A$	$B$	$C$
Male	32	36	12
Female	18	30	12

Test, at the 5% significance level, whether there is a difference between coffee preferences of male and female customers. [8]

A larger random sample is now taken. It consists of  $80n$  male customers and  $60n$  female customers, where  $n$  is a positive integer. It is found that the proportions choosing each brand are identical to those in the smaller sample. Find the least value of  $n$  that would lead to a different conclusion for the 5% significance level hypothesis test. [4]

11 Answer only **one** of the following two alternatives.

**EITHER**

A smooth sphere, with centre  $O$  and radius  $a$ , is fixed on a smooth horizontal plane  $\Pi$ . A particle  $P$  of mass  $m$  is projected horizontally from the highest point of the sphere with speed  $\sqrt{\left(\frac{2}{5}ga\right)}$ . While  $P$  remains in contact with the sphere, the angle between  $OP$  and the upward vertical is denoted by  $\theta$ . Show that  $P$  loses contact with the sphere when  $\cos \theta = \frac{4}{5}$ . [6]

Subsequently the particle collides with the plane  $\Pi$ . The coefficient of restitution between  $P$  and  $\Pi$  is  $\frac{5}{9}$ . Find the vertical height of  $P$  above  $\Pi$  when the vertical component of the velocity of  $P$  first becomes zero. [6]

**OR**

A factory produces bottles of spring water. The manager decides to assess the performance of the two machines that are used to fill the bottles with water. He selects a random sample of 60 bottles filled by the first machine  $X$  and a random sample of 80 bottles filled by the second machine  $Y$ . The volumes of water,  $x$  and  $y$ , measured in appropriate units, are summarised as follows.

$$\Sigma x = 58.2 \quad \Sigma x^2 = 85.8 \quad \Sigma y = 97.6 \quad \Sigma y^2 = 188.6$$

A test at the  $\alpha\%$  significance level shows that the mean volume of water in bottles filled by machine  $X$  is less than the mean volume of water in bottles filled by machine  $Y$ . Find the set of possible values of  $\alpha$ . [12]



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